On the leptonic partial widths of the excited ψ states

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The resonance parameters of the excited ψ-family resonances, namely the $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$, were determined by fitting the R-values measured by experiments. It is found that the previously reported leptonic partial widths of these states were merely one possible solutions among a four-fold ambiguity. By fitting the most precise experimental data on the R-values measured by the BES collaboration, this work presents all four sets of solutions. These results may affect the interpretation of the charmonium and charmonium-like states above 4 GeV/c^2.

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I. INTRODUCTION

Charmonium, bound state of a charm and an anti-charm quarks, is one of the most interesting two-body systems which were studied extensively in particle physics. Although the first charmonium state was discovered more than thirty-five years ago, there are still many puzzles in charmonium physics. The charmonium spectroscopy below the open charm threshold has been well measured and agrees with the theoretical expectations (such as potential models and lattice QCD); however, for the charmonium states above the open charm threshold, there are still lack of adequate experimental information and solid theoretical inductions. For example, recently many new particles have been discovered, named XYZ-particles, and the overwhelming vector states in the 4 GeV/c^2 to 5 GeV/c^2 mass range make the classification of these states as the charmonia questionable [1–3]. In explaining these vector charmonium states, the leptonic partial widths provide very important information. As we know, the vector quarkonium states could be either S-wave or D-wave spin-triplet states, with the S-wave states couple strongly to lepton pair while the D-wave states couple weakly since the latter are only proportional to the second derivative of the wave-function at the origin squared, as expected in the potential models. This leads people believe that the $\psi(4040)$ is the $3S$ charmonium state, $\psi(4160)$ the $2D$ state, and $\psi(4415)$ the $4S$ state. This has been a well accepted picture for more than two decades before the discovery of the so-called Y particles, namely, the $Y(4008)$ and $Y(4260)$ observed in $e^+e^- \to \pi^+\pi^- J/\psi$ final state [4, 5], and the $Y(4360)$ and $Y(4660)$ observed in $e^+e^- \to \pi^+\pi^-\psi'$ final state [6, 7]. With seven states observed between 4.0 GeV/c^2 and 4.7 GeV/c^2, some people started to categorize some of these as non-conventional quarkonium states, while others tried to accommodate all of them in modified potential models. Many of the theoretical models use the leptonic partial widths of these states to distinguish them between S- and D-wave assumptions [8, 9], and most of the time, the values on the leptonic partial widths are cited from the PDG [10] directly. Although the resonance parameters of these excited ψ states have been measured by many experimental groups, all of them were obtained by fitting the R-values measured in the relevant energy region. The most recent ones, which were from a sophisticated fit to the most precise R-values measured by the BES collaboration [11, 12], are the only source of the leptonic partial widths of these three ψ states now quoted by the PDG [10].

In fitting to the BES data, unlike the previous analyses, the BES collaboration considered the interference between the three resonances decaying into the same final modes, and introduced a free relative phase for the amplitude of each resonance [13]. The new parametrization of the hadronic cross section results in a prononce increase of the $\psi(4160)$ mass, and significant decrease of the leptonic partial widths of $\psi(4160)$ and $\psi(4415)$.

As has been pointed out in a recent study [14], there are multiple solutions in fitting one dimensional distribution with the coherent sum of several amplitudes and free relative phase between them. Exactly the same kind of fit to the R-value is performed in the present study, multiple solutions are indeed found in extracting the resonance parameters of the excited ψ states. The only difference between these multiple solutions is the coupling to the e^+e^-, namely the leptonic partial width, while the masses and the widths of the resonances remain the same for all the solutions.

In the following, we firstly introduce a simplified fit scheme similar to that used by the BES collaboration, and extract the resonance parameters. Then we study the multiple solution problem in the light of the toy simulated data for illustrative purpose. We will discuss the consequence of the multiple solutions in fitting the R-values at the end of the paper.

II. THE FIT TO THE R DATA

To faciliate our study, only the R-values provided by the BES collaboration are used [11, 12], and in the data

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fitting, only statistical uncertainties are considered, as the systematic errors at all the energy points are highly correlated.

We fit the e^+e^- annihilation cross section $\sigma(e^+e^-) = R \cdot 86.85/s$ (s in GeV^2 and σ in nb). The standard chisquare estimator is constructed as

$$\chi^2 = \sum_{l=1}^{N_{pt}} \frac{(\sigma_l^{exp.} - \sigma_l^{the.})^2}{(\Delta\sigma_l^{exp.})^2} , \qquad (1)$$

where $\sigma_l^{exp.}$ and $\Delta\sigma_l^{exp.}$ indicate respectively the experimentally measured cross section and its error at the l-th energy point (the number of points is denoted as N_{pt}), while $\sigma_l^{the.}$ is the corresponding theoretical expectation, which is composed of two parts

$$\sigma^{the.}(s) = \sigma^{res.}(s) + \sigma^{con.}(s) . \qquad (2)$$

Here $\sigma^{con.}$ is the contribution from continuum and is parameterized simply as

$$\sigma^{con.}(s) = A + B(\sqrt{s} - 2M_{D^\pm}) , \qquad (3)$$

where A and B are free parameters, and $M_{D^\pm}$ is the mass of the charged D meson. $\sigma^{res.}$ is the contribution from the resonances. Here following previous analyses [13, 15], the assumption that the continuum production and the resonance decays don't interfere with each other is adopted. The three wide resonances shown in the data are close and have the same decay modes, the interferences between them must be included. Then the amplitude reads

$$T_j(s) = \frac{\sqrt{12\pi\Gamma_j^h\Gamma_j^{ee}}\, e^{i\phi_j}}{s - M_j^2 + iM_j\Gamma_j^t} , \qquad (4)$$

with Γ_j^t, Γ_j^h, Γ_j^{ee}, and M_j denoting total width, partial width to hadrons, partial width to e^+e^- pair, and mass of the resonance j, respectively. The total amplitude, which is the coherent sum of the three resonances, once squared, contains the interferences of the type $\Re T_i^* T_j$. Here ϕ_j is a phase associated to resonance j. If the resonances are quite broad, the interference effect will distort the shape of the resonances, the width might appear broader or narrower, and the position of the peak might be shifted as well.

The total cross section of the resonances

$$\sigma^{res.}(s) = \left| \sum_{j=1}^{3} T_j(s) \right|^2 . \qquad (5)$$

Since what we actually obtain is the squared module of the amplitude, only two relative phases are relevant.

By minimizing the χ^2 defined in Eq.(1), we obtain the results as displayed in Fig. 1. Just as expected, the interference effect changes the shape of each resonance significantly: the $\psi(4040)$ becomes narrower while the $\psi(4160)$

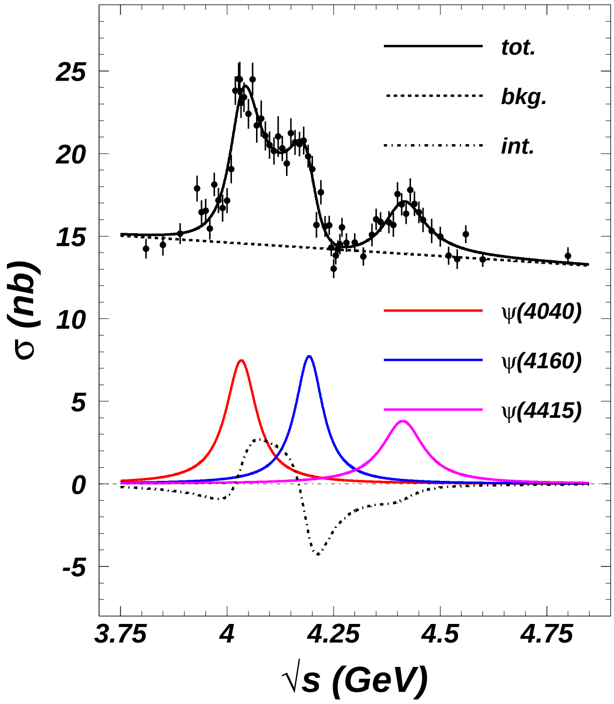

FIG. 1: Total hadronic cross section in nb obtained as $\sigma(e^+e^- \to \text{hadrons}) = R \cdot 86.85/s$ (s in GeV^2) from R measurements of BES [11, 12] and the fit to data. The curves show the best fit (identical for all the four solutions), the contribution from each resonance, as well as the interference term (one of the four solutions).

and $\psi(4415)$ become wider than the previous published results where the interferences between resonances are neglected [15].

The fit results are presented in Table I, there are four solutions found in the fit. It should be noted that the four solutions have identical χ^2, masses, and total widths for the resonances, but different partial widths to lepton pairs. One may suspect that the existence of four solutions is due to low precision of the measurements. We will show in the next session that multiple solution is a real effect. The improvement of the precision of the measurements can not change the fact that there are four solutions in fitting these data.

III. MULTIPLE SOLUTIONS

The existence of four solutions in the fit described above can be tested through the toy simulated data. The special steps for this approach is:

1. The curve of the best fit in previous session (with background contribution subtracted) is used as the probability density function (PDF) to compute a set of experimental points at different energies ($N_{pt} = 100$ in this work) within the range 3.75 GeV $\leq \sqrt{s} \leq$ 4.85 GeV.

TABLE I: Four groups of solutions for the data fitting. The four solutions have identical resonance masses (M) and total widths (Γ_t), but significant different leptonic partial widths (Γ_{ee}) and the relative phases (ϕ). The fit yields $\chi^2 = 91$, $A = (15.05 \pm 0.59)$ nb, and $B = (-1.64 \pm 0.67)$ nb/GeV for all the solutions.

Parameter	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV)	4034 ± 6	4193 ± 7	4412 ± 15
Γ_t (MeV)	87 ± 11	79 ± 14	118 ± 32
$\Gamma_{ee}^{(1)}$ (keV)	0.66 ± 0.22	0.42 ± 0.16	0.45 ± 0.13
$\phi^{(1)}$ (radian)	0 (fixed)	2.7 ± 0.8	2.0 ± 0.9
$\Gamma_{ee}^{(2)}$ (keV)	0.72 ± 0.24	0.73 ± 0.18	0.60 ± 0.25
$\phi^{(2)}$ (radian)	0 (fixed)	3.1 ± 0.7	1.4 ± 1.2
$\Gamma_{ee}^{(3)}$ (keV)	1.28 ± 0.45	0.62 ± 0.30	0.59 ± 0.20
$\phi^{(3)}$ (radian)	0 (fixed)	3.7 ± 0.4	3.8 ± 0.8
$\Gamma_{ee}^{(4)}$ (keV)	1.41 ± 0.12	1.10 ± 0.15	0.78 ± 0.17
$\phi^{(4)}$ (radian)	0 (fixed)	4.1 ± 0.1	3.2 ± 0.3

TABLE II: Fit results for four groups of solutions with the toy simulated data points. The definitions of the parameters are the same as in Table I. The fit yields $\chi^2 = 1.0 \times 10^{-3}$ for all the solutions with the expectation of $\chi^2 = 0$.

Parameter	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV)	4033.5 ± 0.3	4192.8 ± 0.3	4412.4 ± 0.4
Γ_t (MeV)	87.23 ± 0.49	79.00 ± 0.53	118.11 ± 0.56
$\Gamma_{ee}^{(1)}$ (keV)	0.664 ± 0.005	0.417 ± 0.004	0.454 ± 0.003
$\phi^{(1)}$ (radian)	0 (fixed)	2.701 ± 0.012	2.002 ± 0.012
$\Gamma_{ee}^{(2)}$ (keV)	0.723 ± 0.006	0.731 ± 0.005	0.596 ± 0.003
$\phi^{(2)}$ (radian)	0 (fixed)	3.051 ± 0.001	1.432 ± 0.014
$\Gamma_{ee}^{(3)}$ (keV)	1.283 ± 0.005	0.620 ± 0.006	0.590 ± 0.003
$\phi^{(3)}$ (radian)	0 (fixed)	3.732 ± 0.006	3.789 ± 0.013
$\Gamma_{ee}^{(4)}$ (keV)	1.397 ± 0.006	1.087 ± 0.008	0.774 ± 0.003
$\phi^{(4)}$ (radian)	0 (fixed)	4.082 ± 0.005	3.218 ± 0.009

2. A relative error of 1% and an absolute error of 0.01 nb are added in quadrature and the total error is assigned to each data point. The inclusion of a 0.01 nb absolute error is to weaken the chisqure-weight of points with small cross sections.

Fit the simulated data with the similar χ^2 in Eq. (1) but with only resonance cross section included (i.e., $\sigma^{the.}(s) = \sigma^{res.}(s)$ in Eq. (2)), the four groups of solutions obtained are summarized in Table II and displayed in Fig. 2. Comparison of two tables obviously indicates that the central values of the parameters are consistent with each other.

From Table II and Fig. 2, we can see that the largest Γ_{ee} is more than twice the smallest value in the four solutions. We also notice that the first solution is identical to the BES published one [13], and the second solution is about the same as the one listed as the best estimation

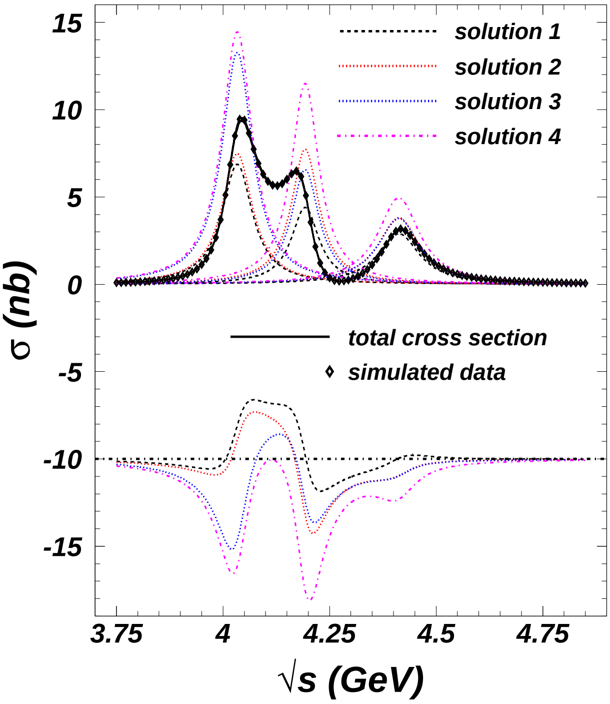

FIG. 2: Four groups of solutions obtained from the fit to the generated data. For clearness, the interference curves have been moved downward by 10 nb, the black dashed line at -10 nb corresponds to zero cross section.

of the partial widths of these states by the PDG [10].

IV. DISCUSSIONS

As the Γ_{ee} of the vector resonances are closely related to the nature of these states, the choice among the distinctive solutions affects the classification of the charmonium and charmonium-like states observed in this energy region.

The calculation of the e^+e^- partial widths of the S-wave charmonium states is well summarized recently in Table 2 of Ref. [16]. We can see clearly that many of the theoretical calculations give large $\psi(3S)$ and $\psi(4S)$ decay widths compared to the PDG values (about the same as the second solution listed in Table I above), but the agreement with the third or the fourth solution is much better.

The $Y(4260)$ was proposed to be the $\psi(4S)$ state and the $\psi(4415)$ be $\psi(5S)$ in Refs. [8, 9], we can see that in this assignment, the calculated partial widths of $\psi(4040) = \psi(3S)$ and $\psi(4415) = \psi(5S)$ [9] agree well with the fourth solution listed in Table I.

Of course the possible mixing between S- and D-wave states will change significantly the theoretical predictions of the partial widths of these states [17], and the QCD correction, which is not well handled [16], may also change the theoretical predictions significantly. So far, we have no concrete criteria to choose any one of the

solutions as the physics one.

It should be noticed that if the Y states are considered together with the excited ψ states in fitting the R-values, there could be even more solutions, the situation may become more complicated.

We also notice that the existence of the multiple solutions is due to the inclusion of a free phase between two resonances, if these phases can be determined by other means (either theoretically or experimentally), it will be very helpful to know which solution corresponds to the real physics.

V. SUMMARY

Based on the R scan data, the resonance parameters of excited ψ-family resonances are fitted. We found that there are four sets of solutions with exactly the same fit quality extracted from the experimental data, but the leptonic partial widths among different sets of solutions differentiate from each other significantly. New information is needed to determine which solution corresponds to the real physics.

Acknowledgments

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